Handling Uncertain Data in Subspace Detection

– Supplementary Material A –
Notational Convention

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A.1 Description

This document presents the symbols used in our paper. The symbols are divided into groups according to the sections that follow. Section A.2 presents the spaces. Sections A.3 and A.4 present the notational convention for elements and operations from geometric algebra and statistics, respectively. The convention for elements and operations from set theory is presented in Section A.5. Well known operations are listed into Section A.6, and the conventions for the proposed approach are presented in Section A.7.

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A.2 Spaces

$\mathbb{Z}$	Set of all integer numbers.
$\mathbb{R}$	Set of all real numbers.
$\mathbb{R}^n$	n-Dimensional vector space over $\mathbb{R}$.
$\bigwedge \mathbb{R}^n$	Multivector space built from $\mathbb{R}^n$.
$\bigwedge^k \mathbb{R}^n$	The part of $\bigwedge \mathbb{R}^n$ in which k-blades reside.
$\mathbb{A}^m$	m-Dimensional affine space.
$\mathbb{P}^m$	m-Dimensional parameter space.
$G(k,n)$	Grassmannian, the set of all k-dimensional linear subspaces of $\mathbb{R}^n$.

A.3 Geometric Algebra

ϕ, θ	Angle in radians.
α, β, etc.	General scalar value (0-blade).
x, y, etc.	General scalar value (0-blade), typically point coordinates.
$\mathbf{a}$, $\mathbf{b}$, etc.	General vector (1-blade).
$\mathbf{A}_{\langle k \rangle}$, $\mathbf{B}_{\langle k \rangle}$, etc.	General k-blade, for $0 \leq k \leq n$, defined as the outer product of k vectors.
$\boldsymbol{A}$, $\boldsymbol{B}$, etc.	General rotor, defined as the geometric product of an even number of unit invertible vectors.
A, B, etc.	General multivector, not necessarily a blade or a rotor.
$\mathbf{e}_i$	i-th Unit basis vector, typically in an orthonormal basis.
$\mathbf{I}_{\langle n \rangle}$	Unit pseudoscalar of a n-dimensional space, defined as $\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \cdots \wedge \mathbf{e}_n$.
$\langle A \rangle_k$	Retrieves the k-dimensional part of A.
$A\,B$	Geometric product of A and B.
$A\,/\,B$	Inverse geometric product, defined as $A\,B^{-1}$.
Q	Scalar-valued function defining a metric on $\mathbb{R}^n$.
$\mathbf{a} \cdot \mathbf{b}$	Vector inner product, defined as $\mathsf{Q}\,(\mathbf{a},\mathbf{b})$.
$\mathbf{A}_{\langle r \rangle} * \mathbf{B}_{\langle s \rangle}$	Scalar product, defined as $\left\langle \mathbf{A}_{\langle r \rangle}\,\mathbf{B}_{\langle s \rangle} \right\rangle_0$.
$\mathbf{A}_{\langle r \rangle} \rfloor \mathbf{B}_{\langle s \rangle}$	Left contraction, defined as $\left\langle \mathbf{A}_{\langle r \rangle}\,\mathbf{B}_{\langle s \rangle} \right\rangle_{s-r}$.

$\mathbf{A}_{\langle r \rangle} \wedge \mathbf{B}_{\langle s \rangle}$	Outer product, defined as $\left\langle \mathbf{A}_{\langle r \rangle} \, \mathbf{B}_{\langle s \rangle} \right\rangle_{r+s}$.
$\mathbf{A}_{\langle r \rangle} \vee \mathbf{B}_{\langle s \rangle}$	Regressive product, defined as $(\mathbf{B}^{*}_{\langle s \rangle} \wedge \mathbf{A}^{*}_{\langle r \rangle})^{-*}$.
$\mathbf{A}_{\langle r \rangle} \cap \mathbf{B}_{\langle s \rangle}$	Meet, defined as the subspace shared by $\mathbf{A}_{\langle r \rangle}$ and $\mathbf{B}_{\langle s \rangle}$.
$\mathbf{A}_{\langle r \rangle} \cup \mathbf{B}_{\langle s \rangle}$	Join, defined as the subspace spanned by the disjoint and by the common parts of $\mathbf{A}_{\langle r \rangle}$ and $\mathbf{B}_{\langle s \rangle}$.
$\widetilde{\mathbf{A}}_{\langle k \rangle}$	Reverse, defined as $(-1)^{k(k-1)/2} \, \mathbf{A}_{\langle k \rangle}$.
$\mathbf{A}^{-1}_{\langle k \rangle}$	Inverse of $\mathbf{A}_{\langle k \rangle}$, defined as $\widetilde{\mathbf{A}}_{\langle k \rangle} \, / \, \left(\mathbf{A}_{\langle k \rangle} * \widetilde{\mathbf{A}}_{\langle k \rangle}\right)$.
$\mathbf{A}^{*}_{\langle k \rangle}$	Dual of $\mathbf{A}_{\langle k \rangle}$, defined as $\mathbf{A}_{\langle k \rangle} \rfloor \mathbf{I}^{-1}_{\langle n \rangle}$.
$\mathbf{D}^{-*}_{\langle n-k \rangle}$	Undual of $\mathbf{D}_{\langle n-k \rangle}$, defined as $\mathbf{D}_{\langle n-k \rangle} \rfloor \mathbf{I}_{\langle n \rangle}$.

A.4 Statistics

$\underline{x}$, $\underline{y}$, etc.	Random variable (Gaussian distribution is assumed).
$\overline{x}$, $\overline{y}$, etc.	Expectation (mean value) of $\underline{x}$, $\underline{y}$, etc.
σ_x, σ_y, etc.	Standard deviation of $\underline{x}$, $\underline{y}$, etc.
σ^2_x, σ^2_y, etc.	Variance of $\underline{x}$, $\underline{y}$, etc.
$\underline{\mathrm{x}}$, $\underline{\mathrm{y}}$, etc.	Vector-valued random variable (Gaussian distribution is assumed).
$\overline{\mathrm{x}}$, $\overline{\mathrm{y}}$, etc.	Expectation (mean vector) of $\underline{\mathrm{x}}$, $\underline{\mathrm{y}}$, etc.
Σ_{x}, Σ_{y}, etc.	Covariance matrix of $\underline{\mathrm{x}}$, $\underline{\mathrm{y}}$, etc.
α	Significance level.
Φ	Cumulative distribution function of the standard normal distribution.

A.5 Set Theory

$\mathcal{A}$, $\mathcal{B}$, etc.	Set of elements.
$\varnothing$	Empty set.
$\lvert\mathcal{A}\rvert$	Cardinality of $\mathcal{A}$.
$\mathcal{A} \cup \mathcal{B}$	Union of $\mathcal{A}$ and $\mathcal{B}$.
$\mathcal{A} \backslash \mathcal{B}$	Relative complement of $\mathcal{A}$ in $\mathcal{B}$.

A.6 Miscellaneous

$\binom{n}{k}$ Binomial coefficient, defined as $n!/\left(k!\,(n-k)\,!\right)$, for $0 \leq k \leq n$.
$n!$ The factorial of a positive integer n, defined as $\prod_{i=1}^{n} i$.

A.7 Conventions of the Proposed Approach

d The number of dimensions of the base space (*i.e.,* space where the geometric interpretation happens).
n The number of dimensions of the representational space.
p The number of dimensions of the intended subspace, for $0 \leq p \leq n$.
r The number of dimensions of an input subspace from the dataset, for $0 \leq r \leq n$.
m The number of dimensions of the parameter space, defined as $p\,(n-p)$.

θ^t One of the m parameters characterizing a given p-blade. It is defined in the $[-\pi/2, \pi/2)$ range.

$\mathbf{X}_{\langle r \rangle}$ An input subspace from the dataset.
$\mathbf{C}_{\langle p \rangle}$ A p-dimensional subspace related to a given input entry $\mathbf{X}_{\langle r \rangle}$.

$\mathbf{E}_{\langle p \rangle}$ The reference blade for p-dimensional subspaces (equation (21) in the paper).
$\mathbf{P}_{\langle 2 \rangle}^{(t)}$ The plane where the t-th rotation applied on $\mathbf{E}_{\langle p \rangle}$ happens.
$\mathbf{F}_{l}^{(t)}$ A space of possibilities (equation (39) in the paper).

$\boldsymbol{R}_t$ A rotor encoding a rotation of θ^t radians on plane $\mathbf{P}_{\langle 2 \rangle}^{(t)}$ (equation (26) in the paper).
$\boldsymbol{T}$ A rotor computed as the geometric product of a sequence of rotors $\boldsymbol{R}_t$ (equation (19) in the paper).