Analysis of Traffic Behavior in Regular Grid and Real World Networks

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Abstract. Traffic jams are reality in many cities and the current ways to deal with the problem always incur costs for citizens. This paper aims to understand the traffic behavior in a regular grid and in a real world network, with uniform and non-uniform demands and different demand loads. To analyze traffic flow, two metrics are used: waiting vehicles and the correlation ratio between the edge occupation and betweenness centrality. Still, a modification in the betweenness centrality is proposed to understand traffic flow. The results show that the network topology represents an important variable in urban traffic, the demand type is determinant in traffic behavior and the modified betweenness centrality shows high correlation ratio with the edge occupation.

Keywords: Urban Traffic Flow; Centrality Measures; Complex Network

1 Introduction

In modern societies, the urban traffic is a crucial aspect to be considered. The increasing demand allied to networks with limited capacity, make traffic jams a reality, causing countless problems for the population. To deal with this problem, some cities are pricing and restricting the access to certain areas. However, these actions directly affect the infrastructure. Other possibility is based on intelligent solutions, providing information to drivers plan their own routes (e.g., expected travel time or the shortest route).

Still, pricing, restricting access or providing information are efforts to deal with the consequences generated by traffic flow. To deal with the cause, it is important to understand the traffic behavior. In this way, works such as [1] [2] are directing efforts to understand the traffic flow by using and adapting centrality measures, however these adaptations are complex.

The present paper is a step to understand the urban traffic flow. The objective of this paper is to improve the understanding about how the traffic behaves in two different networks, a regular grid and a real world network, with different demand types and demand loads. To perform the analysis, two metrics are used, the number of waiting vehicles and the correlation between roads occupation,

betweenness centrality and a modified betweenness centrality, proposed in this paper.

The paper is organized as follow. The next section reviews works related to traffic flow. Following, in Sect. 3 our approach are presented. Scenarios are described in Sect. 4. Section 5 shows results and their analysis and finally, Sect. 6 discusses several aspects of this work and its future directions, and provides concluding remarks.

2 Related Work

Many efforts are being made to understand and predict traffic flow in urban networks. In this way, some works try to understand the traffic flow using centrality measures. Kazerani and Winter [2] try to understand how the physical network structure, characterized by the betweenness centrality, determines the traffic generated by drivers. Betweenness centrality is based on network shortest paths, since rational drivers will travel the shortest paths, the nodes that have more shortest paths will attract more traffic, thus the betweenness centrality can characterize the traffic flow patterns or traffic density.

This view is supported by evidence reported in the literature, e.g., [3] [4] [5], but, according to Kazerani and Winter [2], some factors may impact these arguments. First, it is known that human act at most bounded rationally [6], and hence, their chosen distance function in determining shortest paths is likely not to be topological (alone). Second, the depicted dynamics of travel behavior cannot be solely determined by the characteristics of a static network. Based on this, the authors try to prove the hypothesis that betweenness centrality of the physical travel network is insufficient to explain traffic flow. They adapt the traditional betweenness centrality for travel networks to capture spatial embedding and dynamics in this measure. After this adaptation, authors compare the traditional betweenness centrality with the adapted betweenness centrality and conclude that even with the adapted betweenness centrality they cannot make a reliable explanation or prediction of the traffic flow.

In a similar way, Gao et al [1] estimate traffic flow using GPS-enabled taxi trajectory data in Ingdao, China, and examine the ability of the betweenness centrality to predict traffic flow. The results show that the betweenness centrality is not good to represent the traffic flow. To understand traffic flow, the authors point out the "gap" between this centrality measure and the traffic flow and began to consider spatial heterogeneity of human activities, to explain the observed traffic flow distribution. After that, the correlation coefficient indicates that the proposed model, which incorporates the geographical constraints and human mobility patterns, can represent the traffic flow.

Freeman et al [7] present a betweenness centrality measure based on Ford and Fulkerson [8] theorem, similar to the Freeman's original betweenness centrality, which differs from the original in two ways. First, this measure is defined for both valued and non-valued graph and second, the computation is based on all independent paths between all pairs of points in the network, not only on

geodesic paths. The measure consists in computing the maximum flow from x_j to x_k that pass to x_i divided by the maximum flow to x_j to x_k , producing values that varies between 0 and 1. The concept of this measure is based on the network flow capacity that does not consider the demand. Due to this fact, only the original betweenness centrality will be analyzed.

On the other hand, some efforts are being made to understand the characteristics of the real world networks, sometimes called complex networks. Complex networks, such as small world network [9] or scale free network [10], lie between two extremes, i.e., random network and regularly connected network like lattices.

The properties on the complex network are revealed mainly by early studies focused on the un-weighted network [9] [10]. Indeed, the original work of small world network [9] [11] deals with the World Wide Web, electrical power grid, relationship among film actors, and neural network of a worm as un-weighted networks. Meanwhile, studies investigating weighted network have begun to appear with [12] [13]. In this way, Majima et al [13] made use the knowledge about complex networks to obtain some important and useful information to design and construct transportations networks. Authors analyze transportations networks of railway, subway and waterbus in Japan and conclude that the transport network achieves the high global efficiency by sacrificing the local efficiency meaning redundancy of networks.

Some development explaining how traffic behaves, are based on complex modification in centrality measures and most of them depends on too much information. Still, most studies directed to complex networks use static metrics to evaluate the network efficiency, not considering the traffic dynamics. Thus, in this paper we propose a simple modification in the betweenness centrality and analyze the traffic in a regular grid and in the main arterials of Porto Alegre, Brazil. Two demand types are considered, a uniform and non-uniform demand. Furthermore, we evaluate the waiting vehicles and the correlation between the edge occupation, the classical betweenness centrality and a modified betweenness centrality.

3 Approach

To analyze traffic and network characteristics, it is necessary to observe all roads and intersections, which can be performed by a microscopic modeling.

The microscopic simulation proposed here was implemented using the traffic simulator called SUMO [14]. The main parameters are: the network G to be simulated, the number |N| of travelers and the origin-destination (OD) matrix that will generate the trips.

The method can be separated in five simple steps. First, the network that will be analyzed is selected. Second, define the OD matrix that will generate the trips (i.e., uniform or non-uniform) is defined, and then the number of travelers based on the network capacity is set. The simulation is started and finally, the results are analyzed.

To analyze the emergent behavior and measure the networks we use two metrics. The first consists in counting the number of waiting vehicles in each time-step. Waiting vehicles are a queue formatted by the drivers that could not start their trips due to jams in the start links (i.e. a number of vehicles that cannot be physically inserted in the correct location due to high traffic). This metric is measured in each time-step, counting the total number of waiting vehicles.

The second metric is based on the betweenness centrality. The betweenness centrality defines the importance of edges (or nodes) based in the number of shortest paths that pass through the edge [15]. Betweenness centrality of a node v is defined in Eq. 1, where σ_{st} is the total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

$$BC(g) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{1}$$

More information about centrality measures in complex networks are found in Costa $et\ al\ [16]$.

As mentioned before, the betweenness centrality may not be suitable to make a reliable explanation of the traffic flow. In this way, we propose a simple modification computing only the path that were used by drivers, i.e. we just considering only the chosen routes, instead of calculating the total number of shortest paths form a node to another. Let $\sigma_{s't'}$ be the total number of paths from node s' to node t' and $\sigma_{s't'}(v')$ is the number of those paths that pass through v' and s', t' and v' belonging to the routes set. The modified betweenness centrality is shown in Eq. 2.

$$MBC(g) = \sum_{s' \neq v' \neq t'} \frac{\sigma_{s't'}(v')}{\sigma_{s't'}}$$
 (2)

After, we use the correlation ratio between the classical betweenness centrality, the modified betweenness centrality proposed in this paper, and edges occupation. The classical betweenness centrality and the modified betweenness centrality have static values but the edge occupation varies during the simulation. This variation generates different correlation ratios, according to the simulation time. In this way, the simulation is divided in four parts and, for each edge we calculate the average occupation and then compare with the classical and modified betweenness centrality to obtain the correlation ratio.

4 Scenarios

In this section we describe the networks, demand types and demand loads. The demand is generated by an OD matrix, which separate network in districts and is the probability of determined district to be origin or destination. In this work, we analyze uniform demand, where every district has the same probability to be origin and destination, and a non-uniform demand, where a few nodes have a high probability to be origin and destination.

The demand load consists in the number of vehicles running in the simulation. We analyze the networks with 20% and 40% of the network capacity. Those values were chosen because they represent the average network occupation, and especially in the non-uniform demand, a value higher than 40% may lead to grid locks, what does represent the reality.

4.1 Grid 6x6

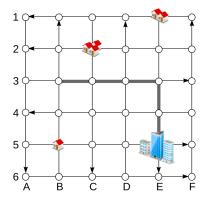


Fig. 1. Grid 6x6

The first network has 36 nodes, arranged as a grid, with 60 links, Figure 1. This network has been studied by Bazzan $et\ al\ [17]$. In this paper, we call this network "Grid 6x6". All links are one-way and drivers can turn in each crossing. Each link has 300m and, due to the fact that each vehicle occupies 5 meters, the network capacity is about 4200 vehicles. Most links have a single lane, that is, may contain 60 vehicles. Five links, however, have three lanes and a capacity of 180 vehicles.

For every driver, its origin and destination are either uniformly selected, or based on an existing (non-uniform) OD matrix. In the former case, we call this uniform demand. For the Grid 6x6, an uniform demand is created by assigning probability of 1/36 to each node being origin and destination. Regarding non-uniform demands, in this paper we use the following. On average, 60% of the drivers have destination at a given link. The other nodes have, each, 1.7% probability of being a destination. Origins are nearly equally distributed in the grid, with three exceptions (three "main residential areas"). The remaining links have each a probability of 1.5%.

4.2 Real World Network

The second network is the main arterials of Porto Alegre, Brazil. In this paper, we call this network "POA-Arterials". The network has 61 nodes with 156 links



Fig. 2. Main Origins and Destinations in Poa Network

totalizing about 212K meters. All links has three lanes, most of them are two-ways, and drivers can turn in each crossing. Due to the fact that each vehicle occupies 5 meters, the network capacity is about 42440 vehicles.

Similarly to the Grid 6x6, for every driver, its origin and destination are either uniformly selected or based on a non-uniform OD matrix. The uniform demand was generated by assigning the probability of 1/61 = 1.64% to every node (both for origin and destination). Regarding non-uniform demands, the origins and destinations are concentrated in 15 main nodes that are depicted in Figure 2. Due to lack of space we do not show the OD matrix but remark that for instance almost 10% of the trips originate in a given node. This is in sharp contrast with the 1.64% in the uniform demand.

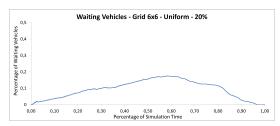
5 Results and Discussions

For better understanding, we analyze waiting vehicles and betweenness centrality correlation separately, forward networks and types of demand.

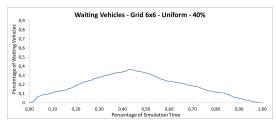
5.1 Waiting Vehicles

The number of waiting vehicles was normalized by the total number of vehicles and the simulation time steps were normalized by the total number of simulation time steps. Figures 3 and 4 show the percentage of waiting vehicles during the simulation for the Grid 6x6. In Figures 3(a) and 3(b) the demand is uniform and the demand load corresponds to 20% and 40% of the network capacity, respectively. In Figures 4(a) and 4(b) the demand is non-uniform with loads of 20% and 40% respectively.

Considering Figures 3(a) and 4(a), the percentage of waiting vehicles are less than 3 percentage points higher with uniform demand. In non-uniform demands, some nodes have higher probability to be origin or destination and, due to the number of vehicles starting their trips in a specific area, larger should be the waiting vehicles queue. A possible explanation is that the demand load is insufficient to cause a large waiting vehicles queue. On the other hand, considering Figures 3(b) and 4(b), the non-uniform demand has a higher number of waiting vehicles, almost 8 percentage points.

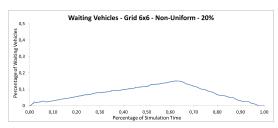


(a) Grid 6x6 with demand load corresponding to 20% of network capacity

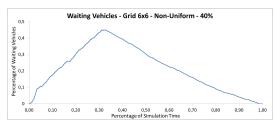


(b) Grid 6x6 with demand load corresponding to 40% of network capacity

Fig. 3. Waiting vehicles in Grid 6x6 with uniform demand



(a) Grid 6x6 with demand load corresponding to 20% of network capacity

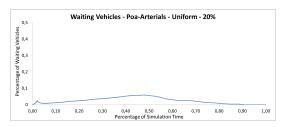


(b) Grid 6x6 with demand load corresponding to 40% of network capacity

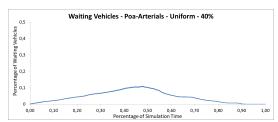
Fig. 4. Waiting vehicles in Grid 6x6 with non-uniform demand

Moreover, in Figures 3(a) and 3(b), when the number of vehicles in simulation is doubled, from 20% to 40%, the percentage of waiting vehicles duplicates too. For the non-uniform demand, Figures 4(a) and 4(b), the percentage of waiting vehicles is three times higher. One possible explanation is that in uniform demands, vehicles tend to start their trips from different places, which does not occur in a non-uniform demand.

It is important to remember that Grid 6x6 does not represent a real world network and the waiting vehicles behavior may be completely different in a real world network. In this way, the following results show the percentage of waiting vehicles during the simulation for the POA-Arterials, Figures 5 and 6. In Figures 5(a) and 5(b) the demand is uniform and the demand load corresponds to 20% and 40% of the network capacity, respectively. In Figures 6(a) and 6(b) he demand is non-uniform with loads of 20% and 40% respectively.



(a) POA-Arterials with demand load corresponding to 20% of network capacity

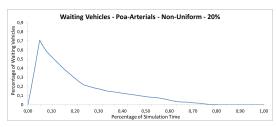


(b) POA-Arterials with demand load corresponding to 40% of network capacity

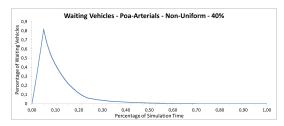
Fig. 5. Waiting vehicles in POA-Arterials with uniform demand

The difference between Figures 5(a) and 5(b) is about 0,05 percentage points and even in Figure 5(b) the maximum percentage of waiting vehicles does not exceed 11%. On the other hand, when the demand is non-uniform, Figure 6, even with loads of 20%, Figure 4(a), the percentage of waiting vehicles exceed 70%. The difference between Figures 6(a) and 6(b) is about 12 percentage points.

The waiting vehicles behavior in Grid 6x6 is different from POA-Arterials, both with uniform and non-uniform demand. In Grid 6x6, the main change in the percentage of waiting vehicles is related with the demand load. For loads of 40%



(a) POA-Arterials with demand load corresponding to 20% of network capacity



(b) POA-Arterials with demand load corresponding to 40% of network capacity

Fig. 6. Waiting vehicles in POA-Arterials with non-uniform demand

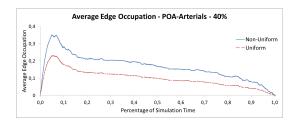
the percentage of waiting vehicles is about 40%, independent the demand type. This behavior cannot be found in POA-Arterials, where the uniform demand with loads of 40% generated a percentage of waiting vehicles about 10% and the non-uniform demand with the same load generated a percentage of waiting vehicles about 80%.

The differences in topology between Grid 6x6 and POA-Arterials lead to different behaviors. In Grid 6x6, vehicles have more possible routes to go from a node s to t than in POA-Arterials. Thus, the demand load is more relevant than demand type in Grid 6x6. On the other hand, analyzing POA-Arterials, the uniform demand for both loads of 20% and 40% generated about 10% of waiting vehicles. When the demand type is non-uniform, the percentage of waiting vehicles is higher than 70%. It means that demand type shows to be more important than the demand loads in POA-Arterials.

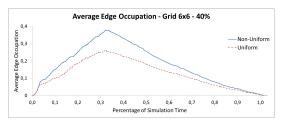
5.2 Betweenness Centrality Correlation

In this section, we analyze the correlation between the edge occupation, the classical betweenness centrality and the modified betweenness centrality. The betweenness centrality computed for edges in Grid 6x6 is the same in every experiment, independent the demand type or demand load. The modified betweenness centrality varies according to demand type and demand load because the metric considers the chosen routes. Still, the edge occupation varies according to the demand type, demand load and the simulation time.

Figure 7 shows the average edge occupation along the simulation. This average was calculated for each time step. Note that the time steps were normalized by the total time steps number. The average edge occupation has practically the same behavior with both loads of 20% and 40%. Thus, Figure 7 shows results for POA-Arterials and Grid 6x6 with uniform and non-uniform demands and loads of 40%.



(a) Edge occupation in POA-Arterials with loads of 40%



(b) Edge occupation in Grid 6x6 with loads of 40%

Fig. 7. Edge Occupation in POA-Arterials and Grid 6x6

The correlation is analyzed in three different simulation stages, the first, second and third quarters (25%) of the total simulation time.

Tables 1 and 2 show the correlation analyses for Grid 6x6 with uniform demand and demand load corresponding to 20% and 40% of the network capacity, respectively. The columns represent the three different periods of the simulation horizon. The same is shown in Tables 3-8.

Metric	Percentage of Simulation Time		
	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.217	0.251	0.226
Correlation of Modified Betweenness			
Centrality and Occupation	0.617	0.635	0.636

Table 1. Grid 6x6 with uniform demand and 20% of the network capacity

Metric	Percentage of Simulation Time		
	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.218	0.249	0.233
Correlation of Modified Betweenness			
Centrality and Occupation	0.624	0.642	0.638

Table 2. Grid 6x6 with uniform demand and 40% of the network capacity

In Tables 1 and 2 it is observed that the classical betweenness centrality shows a small correlation with the occupation. On the other hand, with uniform demand, the modified betweenness centrality show considerable correlation with the edge occupation, in all simulation stages. Still, it is not possible observe a significant difference between Tables 1 and 2. This can be explained because when the demand load increases, the occupation ratio tends to increase uniformly.

Differences in correlations are expected to be more sensible when the demand is non-uniform. Tables 3 and 4 shows the correlation for Grid 6x6 with non-uniform demand and demand load corresponding to 20% and 40% of the network capacity, respectively.

Metric	Percentage of Simulation Time		
MEGLIC	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.115	0.121	0.203
Correlation of Modified Betweenness			
Centrality and Occupation	0.731	0.744	0.738

Table 3. Grid 6x6 with non-uniform demand and 20% of the network capacity

Metric	Percentage of Simulation Time		
Wetric	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.203	0.013	0.146
Correlation of Modified Betweenness			
Centrality and Occupation	0.769	0.775	0.778

Table 4. Grid 6x6 with non-uniform demand and 40% of the network capacity

In Tables 3 and 4, when the demand is non-uniform, the classical betweenness centrality shows worsts correlation compared with the uniform demand. Still, the modified betweenness centrality shows better results when the demand is non-uniform. It is important to remember that Grid 6x6 is a regular grid.

Tables 5 and 6 shows the correlation analyses for POA-Arterials with uniform demand and demand load corresponding to 20% and 40% of the network capacity, respectively.

Metric	Percentage of Simulation Time		
MEGLIC	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.183	0.195	0.155
Correlation of Modified Betweenness			
Centrality and Occupation	0.721	0.753	0.716

Table 5. POA-Arterials with uniform demand and 20% of the network capacity

Metric	Percentage of Simulation Time		
Wetric	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.102	0.093	0.097
Correlation of Modified Betweenness			
Centrality and Occupation	0.782	0.787	0.783

Table 6. POA-Arterials with uniform demand and 40% of the network capacity

In Tables 5 and 6 the classical betweenness centrality has almost no correlation with the edge occupation, i.e., it does not explain the traffic flow. Also, in POA-Arterials, the modified betweenness centrality shows a higher correlation compared to the Grid 6x6.

Finally, Tables 7 and 8 show the most important correlation analyses for POA-Arterials (i.e. real world network) with non-uniform demand (i.e. real world demand) and demand load corresponding to 20% and 40% of the network capacity, respectively.

Metric	Percentage of Simulation Time		
	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.019	0.015	0.020
Correlation of Modified Betweenness			
Centrality and Occupation	0.814	0.827	0.819

Table 7. POA-Arterials with non-uniform demand and 20% of the network capacity

In Tables 7 and 8 the classical betweenness centrality shows no correlation with the edge occupation in any simulation stage. On the other hand, the modi-

Metric	Percentage of Simulation Time		
	0-25%	25%-50%	50%-75%
Correlation of Betweenness			
Centrality and Occupation	0.011	-0.009	0.011
Correlation of Modified Betweenness			
Centrality and Occupation	0.878	0.891	0.885

Table 8. POA-Arterials with non-uniform demand and 40% of the network capacity

fied betweenness centrality shows the highest correlation of all analyzed scenarios.

6 Conclusions

This paper presented an analysis of two different networks, a regular network (Grid 6x6) and a real world network (POA-Arterials) with two demands types (uniform and non-uniform) and two demand loads corresponding to 20% and 40% of the network capacity. The metrics used were the percentage of waiting vehicles and the correlation between edges occupation and classical betweenness centrality and the correlation between modified betweenness centrality and edge occupation.

Waiting vehicles in the POA-Arterials, with non-uniform demand, the percentage of waiting vehicles grows significantly because many drivers wish to start their trips in the same edges. In Grid 6x6, the differences caused by the demand type are smaller than the differences caused by the demand load.

The classical betweenness centrality shows a low correlation with the edge occupation in every case. In the POA-Arterials, the presented behavior was expected, as shown in Sec. 2. However, in the Grid 6x6 the correlation between edges occupation and the betweenness centrality should be higher, especially with uniform demand. The reason may be that, Grid 6x6 is not a perfect regular grid and drivers may have some problems while trying to cross edges with three lanes, due to traffic flow.

The modified betweenness centrality proposed in this paper shows high correlation ratio in all scenarios, especially in POA-Arterials, with non-uniform demand, where the correlation ration reaches 0.891. This happen because the modified betweenness centrality is based on routes chosen by drivers while the classical betweenness centrality take all possible routes into account.

In this work we assumed that routes are completely known but this is not a real assumption. Thus, as future work, it is important to identify the correlation when the origins and destinations are provided, but routes are unknown.

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