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BASICS OF ALGORITHM ANALYSIS

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Outline

1. Stable matchings
2. Representative problems and typical complexities
3. Other problems

Definition

- The problem: n women, n men, each w/ a priority list of the members of the opposite sex. How to match them best? What's best? Simpler: find a *stable* matching.
- Let the women be $w_1, \ldots, w_n$ and the men $m_1, \ldots, m_n$ and let $r(m_i, w_j) \in [n]$ be the rank of woman w_j on m_i's list.
- When is a matching *unstable*?

How to find a stable matching?

```
while not stable: switch
```

Ideias?

The engagement algorithm

Main idea: Each man goes over his candidate list in order, and proposes to the next candidate, and then stops. The engaged pairs marry.

```
while there's a free man m with a candidate do
  m proposes to its (current best) candidate w
  case w is free: (m,w) get engaged
  case w is engaged with m':
    if r(w,m)<r(w,m')
      m' is set free
      (m,w) get engaged
    else
      m remains free
      (m',w) remain engaged
end
return the engaged pairs
```

Complexity?

Complexity

Observations:

- Each proposal (m,w) happens at most once.
- There are at most n^2 pairs.
- Thus: the algorithm terminates in $O(n^2)$ iterations.

Cost of an iteration?

Correctness

More observations

- There exist several stable matchings
- A pair (m,w) is *valid* if there's some stable matching containing (m,w)
- For each man m we can define best(m) as the best ranked woman such that (m,best(m)) is valid
- Similarly we can define worst(m), and the same for the women
- Fact: Our algorithm finds always (m,best(m)) for all men, and (worst(w),w) for all women
- If the women propose, this is inverted

Binary search

Weighted interval scheduling

Bipartite matching

Independent set