

Solução da prova de recuperação

Questão 0.1

- (a) $\forall y f(x, y) = x$
 $U = \{a\}, f = \{(a, a) \mapsto a\}, x \mapsto a$
 $U = \{a, b\}, f = \{(a, a) \mapsto b, (a, b) \mapsto b, (b, a) \mapsto b, (b, b) \mapsto b\}, x \mapsto a$
- (b) $\exists x \forall y f(x, y) = y$
 $U = \{a\}, f = \{(a, a) \mapsto a\}$
 $U = \{a, b\}, f = \{(a, a) \mapsto b, (a, b) \mapsto b, (b, a) \mapsto b, (b, b) \mapsto b\}$
- (c) $\exists x (P(x) \wedge \forall y P(f(x, y)))$
 $U = \{a\}, P = U, f = \{(a, a) \mapsto a\}$
 $U = \{a\}, P = \emptyset, f = \{(a, a) \mapsto a\}$

Questão 0.2

- (a) $\forall x x > 0 \rightarrow \exists y y > 0 \wedge y \times y = x$ (com $x > y := \neg(x < y) \wedge \neg(x = y)$).
- (b) $M \rightarrow I$, com $M := \forall x \forall y x < y \rightarrow f(x) < f(y)$ (f é estritamente monótona) e $I := \forall x \forall y f(x) = f(y) \rightarrow x = y$ (f é injetiva).
- (c) $S \wedge I$, com $S := \forall x \exists y f(y) = x$ (f é surjetiva).

Questão 0.3

- (a) $\vdash (\forall x P(x)) \rightarrow (\exists x P(x))$

1	$\forall x P(x)$	hipótese
2	$P(y)$	$\forall x e1$
3	$\exists x P(x)$	$\exists x i2$
4	$(\forall x P(x)) \rightarrow (\exists x P(x))$	$\rightarrow_i 1-3$

- (b) $\vdash (\exists x P(x) \wedge Q(x)) \rightarrow (\exists x P(x)) \wedge (\exists x Q(x))$

1	$\exists x P(x) \wedge Q(x)$	hipótese
2	$x_0 \quad P(x_0) \wedge Q(x_0)$	hipótese
3	$P(x_0)$	$\wedge_{e_1} 2$
4	$Q(x_0)$	$\wedge_{e_2} 2$
5	$\exists x P(x)$	$\exists x i3$
6	$\exists x Q(x)$	$\exists x i4$
7	$(\exists x P(x)) \wedge (\exists x Q(x))$	$\wedge_i 5,6$
8	$(\exists x P(x)) \wedge (\exists x Q(x))$	$\exists x i1,2-7$
9	$(\exists x P(x) \wedge Q(x)) \rightarrow (\exists x P(x)) \wedge (\exists x Q(x))$	$\rightarrow_i 1-8$

- (c) $\vdash (\exists x \forall y P(x, y)) \wedge (\forall x P(x, x) \rightarrow \exists y Q(y, x)) \rightarrow \exists y \exists x Q(y, x)$

1	$(\exists x \forall y P(x, y)) \wedge (\forall x P(x, x) \rightarrow \exists y Q(y, x))$	hipótese
2	$\exists x \forall y P(x, y)$	$\wedge_{e_1} 1$
3	$\forall x P(x, x) \rightarrow \exists y Q(y, x)$	$\wedge_{e_2} 1$
4	$x_0 \quad \forall y P(x_0, y)$	hipótese
5	$P(x_0, x_0)$	$\forall x e_4$
6	$P(x_0, x_0) \rightarrow \exists y Q(y, x_0)$	$\forall x e_3$
7	$\exists y Q(y, x_0)$	$\rightarrow_e 5, 6$
8	$y_0 \quad Q(y_0, x_0)$	hipótese
9	$\exists x Q(y_0, x)$	$\exists x i_8$
10	$\exists y \exists x Q(y, x)$	$\exists x i_9$
11	$\exists y \exists x Q(y, x)$	$\exists x e_{7, 8-10}$
12	$\exists y \exists x Q(y, x)$	$\exists x e_{2, 4-11}$
13	$(\exists x \forall y P(x, y)) \wedge (\forall x P(x, x) \rightarrow \exists y Q(y, x)) \rightarrow \exists y \exists x Q(y, x)$	$\rightarrow_i 1-12$

(d) $\exists x P(x) \rightarrow Q \dashv\vdash (\forall x P(x)) \rightarrow Q$.

1	$\exists x P(x) \rightarrow Q$	premissa
2	$\forall x P(x)$	hipótese
3	$x_0 \quad P(x_0) \rightarrow Q$	hipótese
4	$P(x_0)$	$\forall x e_2$
5	Q	$\rightarrow_e 4, 3$
6	Q	$\exists x e_{1, 3-5}$
7	$(\forall x P(x)) \rightarrow Q$	$\rightarrow_i 2-6$
1	$(\forall x P(x)) \rightarrow Q$	premissa
2	$(\forall x P(x)) \vee \neg(\forall x P(x))$	LEM
3	$\neg \exists x P(x) \rightarrow Q$	hipótese
4	x_0	qq. x_0
5	$\neg P(x_0)$	hipótese
6	$P(x_0)$	hipótese
7	\perp	$\neg_e 6, 5$
8	Q	$\perp_e 7$
9	$P(x_0) \rightarrow Q$	$\rightarrow_i 6-8$
10	$\exists x P(x) \rightarrow Q$	$\exists x i_9$
11	\perp	$\neg_e 10, 3$
12	$P(x_0)$	PBC 5-11
13	$\forall x P(x)$	$\forall x e_{4-12}$
14	Q	$\rightarrow_e 13, 1$
15	$P(x)$	hipótese
16	Q	cópia 14
17	$P(x) \rightarrow Q$	$\rightarrow_i 15-16$
18	$\exists x P(x) \rightarrow Q$	$\exists x i_{17}$
19	\perp	$\neg_e 18, 2$
20	$\exists x P(x) \rightarrow Q$	PBC 3-19