ARMAX Model of Elastic Nuclei for Rotors MEMS

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Abstract

This paper proposes a procedure to extract characteristic parameters of MEMS structures, using systems identification. The target structures are the nuclei of micro-rotors and the behavioral performance. The structures are described in 3D solid model and the behavioral data necessary to the extraction of characteristic parameters, used in macro-models, are obtained through numerical simulations based on finite elements method. To parameter extraction were adopted the ARMAX model and the ELS estimator. Experimental data obtained from two topologies of micro-nuclei, with one degree of freedom, demonstrates satisfactory estimate results when applied to analytical models that the extracted parameters, which motivates the application of other techniques available in the systems identification domain.

1. Introduction

Micro Electro-Mechanical Systems are micro-transducers which performs functions of sensing and actuation. Among these, the elastic deformation transducer (micro-nucleus) and electrostatic actuation (comb-drive), because that presents quick answer, low power consumption and easy integration with electronic circuits. The basic operation of these MEMS is based in the principle of resonance frequency, which is definite by geometric forms and material properties of micro-nuclei itself. This type of device when is properly willing can configure: relays, tweezers, oscillators, filters, processors, mixers, gyroscopes, accelerometers and others [1].

The dimensions of micrometer order, the thickness of the device, the lack of understanding of the physical effects of intermolecular forces in these dimensions, the change of material properties of elements when reduced to small scales, are factors that affect the quality of the operating structure as a whole. The research in MEMS device area aimed to reduce costs and confirm the quality of these devices. These factors have been guaranteed in batch production, where millions of components are manufactured in a single layer and tested by sampling. However, the industry currently is interested in testing each manufactured devices. Therefore, detection tests for defects and failures must be optimized [2].

Aiming to overcome the difficulties mentioned has been used system identification as an effective alternative [3]. In the last years, combining “white box” modeling and “black box”modeling has occurred some progress in improving the achievements of the behavioral model of these devices. This combination results in the gray box modeling, meanwhile in the scientific literature has available limited information as to apply this technique in MEMS devices. Therefore, the purpose of this paper consists to use the Autoregressive Moving Average with Exogenous inputsmodel (ARMAX) and Extended Least Squares (ELS) estimator to obtain the linear mathematical model representing the behavioral performance of the micro-nuclei MEMS. The paper is organized as follows. In Section 2 is characterized the gray box technique. The Section 3 shows the application of gray box modeling in micro-nuclei MEMS. The Section 4 presents the results and their discussion. And finally, the Section 5 presents the conclusion and proposal of future work.

2. “Gray box”Modeling

The behavioral performance of the micro-nuclei has been mathematically modeled using techniques “white box” modeling and “black box” modeling. The “gray box” modeling combines the advantages of the models mentioned above. The main points in favor of gray box identifying in the technical literature are: (i) reducing the number of parameters in the models, (ii) greater capacity to reproduce other features besides of the identification data, (iii) greater hardiness, and (iv) more suitable for the development of control systems [4]. Thus, data input and output and a priori information are used to obtain the performance behavior of the device and simultaneously optimize the computational processing time. The signal acquisition and sampling process are illustrated in fig. 1.

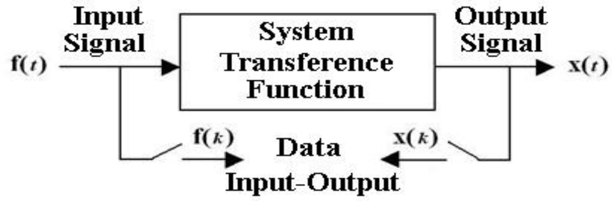
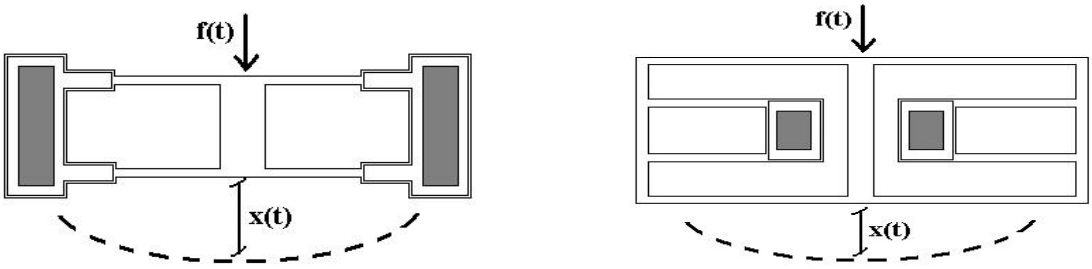


Fig.1. –Gray box systems identification.

3. Obtaining the Behavioral Performance of Micro-Nuclei

The “gray box” modeling follows the identification procedure. The process is divided in five main steps: dynamic tests and data collection, choice of mathematical representation to be used, determination of the model structure, estimation of parameters and model validation [5].

The micro-nuclei are constituted basically of beams and columns, as a non-rigid elements (deformable); and anchors (or inclosings) and masses, as rigid elements (non-deformable). The combination of these elements can generate different topologies of micro-nuclei [6]. The topologies of double bridge and hinge, shown in fig. 2, are the object of study. These structures are applied in medicine and telecommunications respectively, and its choice is based on simplicity and the displacement is unidirectional.



(a) (b)

Fig.2.–Micro-nuclei (a) Double bridge (b) Hinge.

The experimental platform is developed using finite elements method by the computational tool ANSYS. The test signal $f(t)$ is a step with an amplitude of 0.14μN. This signal allows to verify the stability, asymmetrical dynamics and minimum phase of the structure. Its amplitude is selected within a range of values so that both structures do not collapse when the force is applied.

For the sampled signals, $f(k)$ and $x(k)$, retain the fundamental characteristics of the original signal, where the step is a signal without frequency and without energy, the sampling does not follow the Nyquist criteria. Tests performed show the necessity of 300 samples to compare the response of the model to be estimated with the real model. For the discretization of the signs shall be used the Tustin discretization, selected as the experimental tests for data recovery, had a better efficiency when compared to other discretization methods as: backward, forward and zero order hold.

The mathematical representation for the model to be estimated is the ARMAX. This representation belongs to the class of models of error in the equation. Its choice is made based on the difficulty in “gray box” modeling, since they cannot identify all the phenomena that happen in the micrometer scale. In this case the equation error is modeled as a moving average process since this model considers the correlated noise [3]. ARMAX model is presented in equation (1).

$$x(k) = \frac{B(q)}{A(q)} f(k) + \frac{C(q)}{A(q)} \vartheta(k) \tag{1}$$

where $A(q) = a_1 q^{-1} + \cdots a_{nx} q^{-nx}$ and, $B(q) = b_1 q^{-1} + \cdots a_{nf} q^{-nf}$ are polynomials which contains, respectively, the poles and zeros of the system; $C(q) = 1 + c_1 q^{-1} + \cdots a_{nv} q^{-nv}$ is the polynomial which contains the poles and zeros that affect the noise; q is the delay operator; n_x, n_f e n_v, are the longest delays of polynomials $A(q)$, $B(q)$ e $C(q)$ respectively, and k is the discrete instants.

Based on a priori information of the classical analytic model, the representation chosen to describe the performance of micro-nuclei MEMS is second order, which is defined by Ordinary Differential Equation (ODE) non-homogeneous presented in equation (2),

$$M \frac{d^2}{dt^2} x(t) + D \frac{d}{dt} x(t) + K x(t) = f(t) \tag{2}$$

where M is the mass of the spring in kg, being the density of its material (polisilicium) equal $2.33 \times 10^3 \, kg/m^3$. D is the damping caused by the environment in Ns/m, that in this case is the air, whose relative permissivity, viscosity absolute and density are respectively 1.006, $1.8 \times 10^{-5} \, Ns/m^2$ and 1.22 Kg/m^3. K is the coefficient of elasticity in N/m, being 140x10^{9} N/m^{2} the Young module to polisilicium. Finally $f(t)$ is the applied force in N.

The estimation of model parameters is performed by the method ELS in batch. In this process the input data and output are measured and computed at once. The choice is justified by its ease of implementation and efficiency in the estimation of linear systems. Thus, the system is written according to the equation (3),

$$x(k) = \varphi^T \hat{\theta} + e_K \tag{3}$$

where φ is the regressors vector, e_K is the model error, and $\hat{\theta}$ is the vector containing the estimated parameters of micro-nuclei, which are obtained by the equation(4),

$$\hat{\theta} = (\varphi^T \varphi)^{-1} \varphi^T X \tag{4}$$

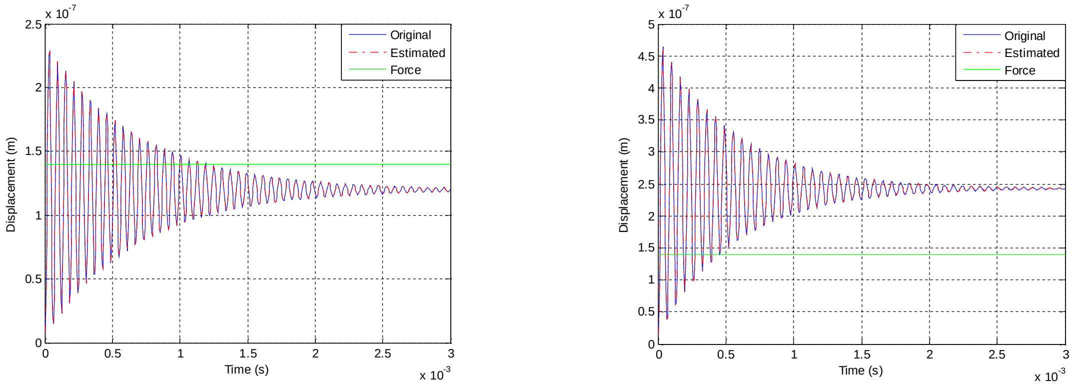
The values of the estimated parameters are shown in Table 2. From the residual error of the parameters and the regressors, a new extended matrix is constructed iteratively. Finally, the model validation is performed by comparing between the dynamics of the model estimated and the real model. The comparison of errors between the test platform and the estimated model, also allows evaluating the efficiency of this model quantitatively. Finally the estimated model is also tested with another excitation signal, known as cross-validation, and the same way compared with the real model.

Table1. Estimated parameters ARMAX

Estimated parameters	**Double bridge**	**Hinge**
$\hat{\theta}_1$	−1,72676	−1,75356
$\hat{\theta}_2$	0,985597	0,981881
$\hat{\theta}_3$	0,056023	0,102349
$\hat{\theta}_4$	0,110733	0,19686
$\hat{\theta}_5$	0,0548917	0,0865737

4. Results and Discussions

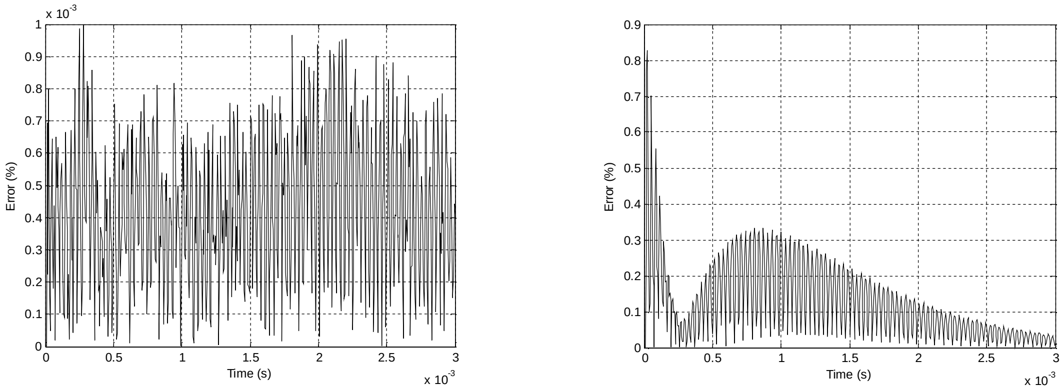
From the comparison between the real performance and the estimated model to both topologies, the input signal in form of step generates an oscillating movement at the start of the displacement featuring the transitional process. For both topologies, the estimated models have a dynamic perfectly compatible with the real dynamics experimental as shown in (fig.3). This fact can be evaluated quantitatively by percent error between the experimental dynamic test platform in relation to the estimated model.



(a) (b)

Fig.3.–Comparison between the dynamics of the real model and estimated model.(a) Double bridge; (b) Hinge.

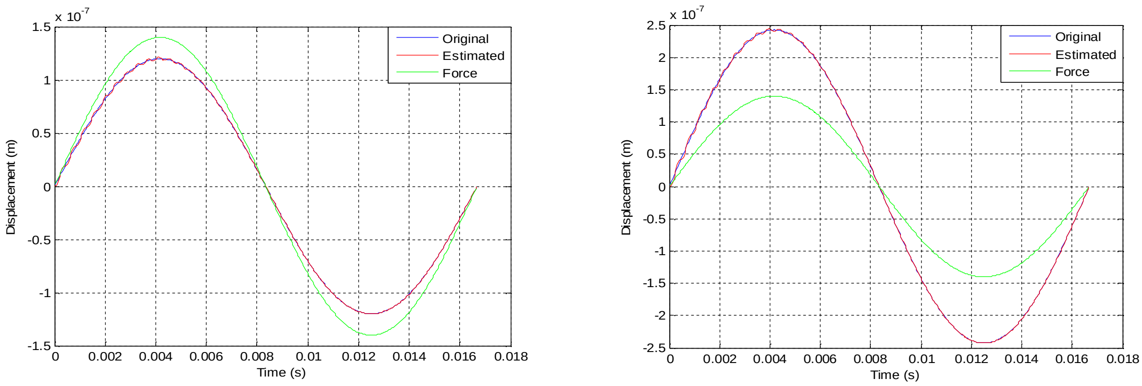
Fig.4 shows how the errors should approach zero. In the case of the hinge profile model of the variations of errors is more evident due to the complexity of the structure although these are very small.



(a) (b)

Fig.4. –Percentage error between real and estimated model. (a) Double bridge; (b) Hinge.

Another way to validate the efficiency of the models is to verify if they respond similarly to the model data measured at different from those used in estimation. Therefore, the responses of micro-nuclei for a sinusoid signal with amplitude of $0.14x10^{-6}N$ are shown in fig.5, therefore, the application of sinusoid signal in structures allows that the values predicted by estimated model and the experimental values satisfy the dynamics of the process.



(a) (b)

Fig.5. –Response of both models subjected to sinusoid. (a) Double bridge, (b) Hinge.

5. Conclusion

In the paper we used the ARMAX model, the ELS estimator and the Tustin discretization to verify the consistency of this new mathematical model to characterize the performance behavior of micro-nuclei MEMS double bridge and hinge types. The “gray box” modeling allows combining the advantages of “black box” and “white box” models as an efficient alternative in identification techniques. In this paper, the coefficients related to the error had no accentuated correlation in the acquisition data. Therefore, it checks the validity of the used technique, being interesting since it allows to obtain the behavioral model of micro-nuclei MEMS without changing the intrinsic properties of the same and the environment in which they are embedded. The mathematical models are considered a non-invasive technique, practical and efficient. The accuracy of the results are satisfactory and similar to those suggested in the literature and should be investigated in detail in future work.

6. References

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