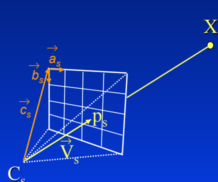
3-D Image Warping

3-D Image Warping [McMillan - Bishop 95]

- Maps images with depth onto arbitrary image planes
- Scene geometry implicitly represented by a pinhole camera and per pixel depth

Demo

3-D Image Warping [McMillan - Bishop 95]



$$\dot{C}_s + t_s\vec{V}_s = X$$

$$\vec{V}_s = \vec{c}_s + p_{sx}\vec{a}_s + p_{sy}\vec{b}_s$$

$$\dot{C}_s + t_s(\vec{c}_s + p_{sx}\vec{a}_s + p_{sy}\vec{b}_s) = X$$

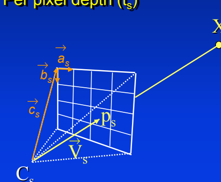
$$\dot{C}_s + t_s M_s p_s = X$$

$$M_s p_s = \begin{bmatrix} a_{si} & b_{si} & c_{si} \\ a_{sj} & b_{sj} & c_{sj} \\ a_{sk} & b_{sk} & c_{sk} \end{bmatrix} \begin{bmatrix} p_{sx} \\ p_{sy} \\ 1 \end{bmatrix}$$

$$t_s = \frac{\|X - C_s\|}{\|\vec{V}_s\|}$$

3-D Image Warping (Cont.)

- What we need to know
  - Camera parameters
    - Position (C_s)
    - Orientation, focal distance, horizontal and vertical FOVs (M_s)
  - Per pixel depth (t_s)



$$\dot{C}_s + t_s M_s p_s = X$$

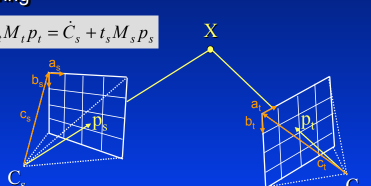
$$\|\vec{a}\| = \frac{\text{width of the projection plane in 3D}}{\text{number of columns in the image}}$$

$$\|\vec{b}\| = \frac{\text{height of the projection plane in 3D}}{\text{number of rows in the image}}$$

3-D Image Warping (Cont.)

- Produces correct 3-D reprojections from arbitrary views
- Compute the projections onto arbitrary image planes by solving

$$\dot{C}_t + t_t M_t p_t = \dot{C}_s + t_s M_s p_s$$



3-D Warping Equation

$$\dot{C}_t + t_t M_t p_t = \dot{C}_s + t_s M_s p_s$$

$$p_t \doteq M_t^{-1}((\dot{C}_s - \dot{C}_t) + t_s M_s p_s)$$

$$p_t \doteq \underbrace{M_t^{-1}(\dot{C}_s - \dot{C}_t)\delta_s}_{epipole} + \underbrace{M_t^{-1} M_s p_s}_{planar\ pespectiveproj}$$

$\delta_s = 1/t_s$ is called the *generalized disparity* of pixel p_s

- 3-D image warping is equivalent to a texture mapping operation followed by a per pixel shift proportional to δ_s in the direction of the epipole
- Generalizes conventional texture mapping

3-D Warping Equation (Cont.)

$$p_t \doteq \underbrace{M_t^{-1}(\dot{C}_s - \dot{C}_t)}_{epipole}\delta_s + \underbrace{M_t^{-1} M_s p_s}_{planar\ pespectiveproj}$$

$$\alpha \begin{bmatrix} u_t \\ v_t \\ 1 \end{bmatrix} = \begin{bmatrix} \vec{a}_t & \vec{b}_t & \vec{c}_t \end{bmatrix}^{-1} \left(\dot{C}_s - \dot{C}_t\right)\delta_s(u_s, v_s) + \begin{bmatrix} \vec{a}_t & \vec{b}_t & \vec{c}_t \end{bmatrix}^{-1} \begin{bmatrix} \vec{a}_s & \vec{b}_s & \vec{c}_s \end{bmatrix} \begin{bmatrix} u_s \\ v_s \\ 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} u_t \\ v_t \\ 1 \end{bmatrix} = \begin{bmatrix} \vec{a}_t & \vec{b}_t & \vec{c}_t \end{bmatrix}^{-1} \begin{bmatrix} \vec{a}_s & \vec{b}_s & \vec{c}_s & \left(\dot{C}_s - \dot{C}_t\right) \end{bmatrix} \begin{bmatrix} u_s \\ v_s \\ 1 \\ \delta(u_s, v_s) \end{bmatrix}$$

3-D Warping Equation (Cont.)

$$u_t = \frac{\vec{a}_s \cdot (\vec{b}_t \times \vec{c}_t)u_s + \vec{b}_s \cdot (\vec{b}_t \times \vec{c}_t)v_s + \vec{c}_s \cdot (\vec{b}_t \times \vec{c}_t) + (\dot{C}_s - \dot{C}_t) \cdot (\vec{b}_t \times \vec{c}_t)\delta(u_s, v_s)}{\vec{a}_s \cdot (\vec{a}_t \times \vec{b}_t)u_s + \vec{b}_s \cdot (\vec{a}_t \times \vec{b}_t)v_s + \vec{c}_s \cdot (\vec{a}_t \times \vec{b}_t) + (\dot{C}_s - \dot{C}_t) \cdot (\vec{a}_t \times \vec{b}_t)\delta(u_s, v_s)}$$

$$v_t = \frac{\vec{a}_s \cdot (\vec{c}_t \times \vec{a}_t)u_s + \vec{b}_s \cdot (\vec{c}_t \times \vec{a}_t)v_s + \vec{c}_s \cdot (\vec{c}_t \times \vec{a}_t) + (\dot{C}_s - \dot{C}_t) \cdot (\vec{c}_t \times \vec{a}_t)\delta(u_s, v_s)}{\vec{a}_s \cdot (\vec{a}_t \times \vec{b}_t)u_s + \vec{b}_s \cdot (\vec{a}_t \times \vec{b}_t)v_s + \vec{c}_s \cdot (\vec{a}_t \times \vec{b}_t) + (\dot{C}_s - \dot{C}_t) \cdot (\vec{a}_t \times \vec{b}_t)\delta(u_s, v_s)}$$

$$r = w_{11}u_s + w_{12}v_s + w_{13} + w_{14}\delta(u_s, v_s)$$

$$s = w_{21}u_s + w_{22}v_s + w_{23} + w_{24}\delta(u_s, v_s)$$

$$t = w_{31}u_s + w_{32}v_s + w_{33} + w_{34}\delta(u_s, v_s)$$

$$u_t = \frac{r}{t} \quad v_t = \frac{s}{t}$$

w_{ij}: recomputed as the target camera parameters change

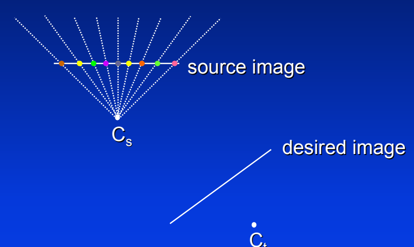
Sub-expressions of *r*, *s* and *t* can be updated incrementally

Disocclusion Artifacts

- Exposures of areas not visible in the original image introduce artifacts
- Need samples from other images to fill the holes

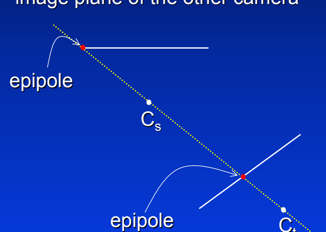
Occlusion-Compatible Order

- List priority algorithm for solving visibility

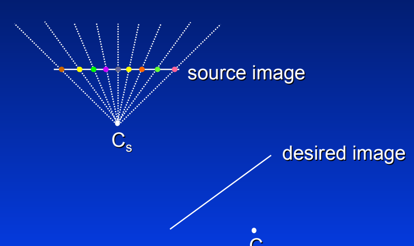


Occlusion-Compatible Order (Cont.)

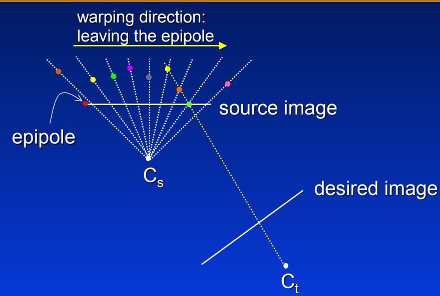
Epipole: projection one center of projection onto the image plane of the other camera



Occlusion-Compatible Order



Occlusion-Compatible Order (Cont.)



Occlusion-Compatible Order (Cont.)

- Algorithm summary

Find the project. of target COP onto the source image plane (epipole)
Divide source image plane into at most 4 sheets based on the epipole
If *target COP* is behind *source COP*
    for each sheet
        warp samples from the epipole towards the borders of the sheet
else
    for each sheet
        warp samples from the borders of the sheet towards the epipole

Computing the Epipole

- Projecting C_t onto the source image plane

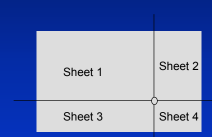
$$\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = M_s^{-1}(\dot{C}_t - \dot{C}_s)$$

- The coordinates of the projected point are given by

$$\bar{e} = \begin{bmatrix} e_x / e_z \\ e_y / e_z \end{bmatrix}$$

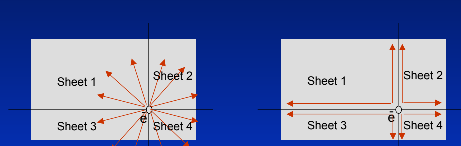
Splitting the Source Image

- The epipole splits the source image in at most 4 sheets



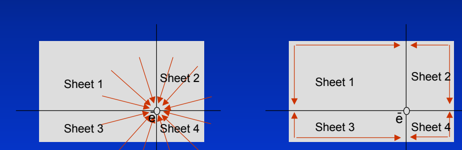
Warping Direction

- e_z negative: target COP behind source COP



Warping Direction

- e_z non-negative: target COP aligned with or in front of the source COP



Warping Images from Real Environments [UNC Chapel Hill]

Reading room: courtesy of the UNC Chapel Hill Image-Based Rendering Group

Depth Acquisition

- Movable cart containing the necessary hardware to acquire range data: rangefinder and panning motor, computer display and B/W monitor (for showing the laser position), PC and battery
- Problems with specular and black surfaces

Courtesy of Lars Nyland, UNC Chapel Hill

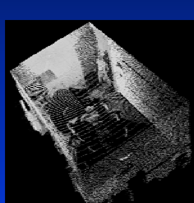
Rangefinder and Panning Motor

- The rangefinder, scanning mirror, and pan-tilt unit.The hi-ball tracking unit is on top, with 6 lenses looking atthe ceiling

Courtesy of Lars Nyland, UNC Chapel Hill

Range Maps

- Examples

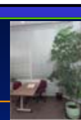


Range Maps (Cont.)

- Warped range map using Multiple-Center-of-Projection Images [Rademacher 98]

Courtesy of Paul Rademacher and Lars Nyland, UNC Chapel Hill

Color Images



- After an environment is scanned, the laser is removed from the panning unit and replaced with the camera
- Camera with mounting bracket to position the COP of the lens at the approximate location as the COP of the rangefinder

Images courtesy of Lars Nyland, UNC Chapel Hill

Color Images (Cont.)

- In order to minimize lens distortions, several overlapping images are taken and only the central portions of the images are used

Courtesy of Lars Nyland, UNC Chapel Hill